EX 25 :

$z_A = 1-i$ et $z_B = 3+i$ $\quad f : M(z) \to M'(z')$ $\quad tq : z' = z^2 - 4z$

1) a)

$A' = f(A) \Leftrightarrow z_{A'} = z_A^2 - z_A \Rightarrow z_{A'} = (1-i)^2 - 4(1-i) \Rightarrow \boxed{z_{A'} = -4+2i}$

$B' = f(B) \Leftrightarrow z_{B'} = z_B^2 - 4z_B \Leftrightarrow z_{B'} = (3+i)^2 - 4(3+i) \Rightarrow \boxed{z_{B'} = -4+2i}$ (*Rque* : $A' = B'$)

b) Soit $M_1(z_1)$ et $M_2(z_2)$

$f(M_1) = f(M_2) \Leftrightarrow z_1^2 - 4z_1 = z_2^2 - 4z_2 \Leftrightarrow (z_1^2 - z_2^2) - 4(z_1 - z_2) = 0$

$\Leftrightarrow (z_1 - z_2)(z_1 + z_2) - 4(z_1 - z_2) = 0 \Leftrightarrow (z_1 - z_2)(z_1 + z_2 - 4) = 0$

$\Leftrightarrow z_1 - z_2 = 0$ *ou* $z_1 + z_2 = 4 \Leftrightarrow z_1 = z_2$ *ou* $\frac{z_1 + z_2}{2} = 2$

Soit « L » le point d'affixe 2.

$f(M_1) = f(M_2) \Leftrightarrow z_1 = z_2$ *ou* $\frac{z_1 + z_2}{2} = z_L$

$\Leftrightarrow M_1 = M_2$ *ou* $L = M_1 * M_2$

$\Leftrightarrow M_1 = M_2$ *ou* $M_2 = S_L(M_1)$

2) $z_I = -3$

a) OMIM' est un parallélogramme ssi

$\overrightarrow{OM'} = \overrightarrow{MI} \Leftrightarrow z' = -3 - z \Leftrightarrow z^2 - 4z = -3 - z \Leftrightarrow z^2 - 3z + 3 = 0$

b) $z^2 - 3z + 3 = 0$

$\Delta = 9 - 12 = -3 \quad \rightarrow \quad \delta = i\sqrt{3}$

Les solutions sont : $z' = \frac{3 - i\sqrt{3}}{2}$ et $z'' = \frac{3 + i\sqrt{3}}{2}$

3)

a) $z' + 4 = z^2 - 4z + 4 = (z-2)^2 \Rightarrow |z' + 4| = |z - 2|^2$ et $Arg(z' + 4) \equiv 2Arg(z - 2)(2\pi)$

$z_J = 2$; $z_K = -4$

b) $M \in C(J, 2) \Rightarrow JM = 2 \Rightarrow |z - 2| = 2 \Rightarrow |z' + 4| = 4 \Rightarrow KM' = 4 \Rightarrow M' \in C(K, 4)$

c) $z_E = -4 - 3i \Rightarrow z_E + 4 = -3i \Rightarrow (z_E + 4) = 3(\cos(-\frac{\pi}{2}) + i.\sin(-\frac{\pi}{2}))$

Qarray.tn Aqra Blech- Qarray.tn